

Hong Kong Mathematics Olympiad (2013 / 2014)

Final Event 1 (Group)

香港數學競賽 (2013 / 2014)

決賽項目 1 (團體)

除非特別聲明，答案須用數字表達，並化至最簡。

Unless otherwise stated, all answers should be expressed in numerals in their simplest forms.

1. 若一個等腰三角形對應底邊（不是兩條等腰邊）的高是 8，且周長是 32，求該三角形的面積。
If an isosceles triangle has height 8 from the base, not the legs, and perimeters 32, determine the area of the triangle.

2. 若 $f(x) = \frac{\left(x + \frac{1}{x}\right)^6 - \left(x^6 + \frac{1}{x^6}\right) - 2}{\left(x + \frac{1}{x}\right)^3 + \left(x^3 + \frac{1}{x^3}\right)}$ 當中 x 是一個正實數，求 $f(x)$ 的最小值。

If $f(x) = \frac{\left(x + \frac{1}{x}\right)^6 - \left(x^6 + \frac{1}{x^6}\right) - 2}{\left(x + \frac{1}{x}\right)^3 + \left(x^3 + \frac{1}{x^3}\right)}$ where x is a positive real number, determine the minimum value of $f(x)$.

3. 求 81 位數 $\overline{111\dots1}$ 除以 81 的餘數。
Determine the remainder of the 81-digit integer $\overline{111\dots1}$ divided by 81.

4. 給定一實數數列 a_1, a_2, a_3, \dots ，它滿足

1) $a_1 = \frac{1}{2}$ ，及

2) 對 $k \geq 2$ ，有 $a_1 + a_2 + \dots + a_k = k^2 a_k$ 。

求 a_{100} 的值。

Given a sequence of real numbers a_1, a_2, a_3, \dots that satisfy

1) $a_1 = \frac{1}{2}$, and

2) $a_1 + a_2 + \dots + a_k = k^2 a_k$, for $k \geq 2$.

Determine the value of a_{100} .

Hong Kong Mathematics Olympiad (2013 / 2014)

Final Event 2 (Group)

香港數學競賽 (2013 / 2014)

決賽項目 2 (團體)

除非特別聲明，答案須用數字表達，並化至最簡。

Unless otherwise stated, all answers should be expressed in numerals in their simplest forms.

1. 若在 $\frac{1}{2} + \frac{1}{4} + \frac{1}{6} + \frac{1}{8} + \frac{1}{10} + \frac{1}{12}$ 中刪去若干項後剩 1，求刪去各項的乘積。

By removing certain terms from the sum, $\frac{1}{2} + \frac{1}{4} + \frac{1}{6} + \frac{1}{8} + \frac{1}{10} + \frac{1}{12}$, we can get 1. What is the product of the removed term(s)?

2. 若 $S_n = 1 - 2 + 3 - 4 + \cdots + (-1)^{n-1}n$ ，當中 n 是正整數，求 $S_{17} + S_{33} + S_{50}$ 的值。

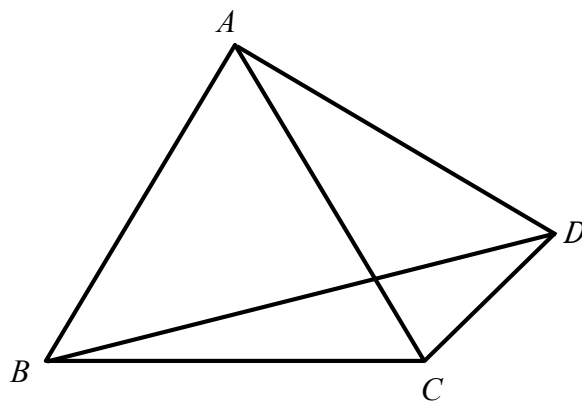
If $S_n = 1 - 2 + 3 - 4 + \cdots + (-1)^{n-1}n$, where n is a positive integer, determine the value of $S_{17} + S_{33} + S_{50}$.

3. A, B, C, D, E 和 F 六人根據英文字母的順序輪班工作。 A 在第一個星期日當值，然後 B 在星期一當值，如此類推。 A 於第 50 個星期的哪一天當值？（答案以數字 0 代表星期日，數字 1 代表星期一，……，數字 6 代表星期六）。

Six people A, B, C, D, E and F are to rotate for night shifts in alphabetical order with A serving on the first Sunday, B on the first Monday and so on. In the fiftieth week, Which day does A serve on? (Represent Sunday by 0, Monday by 1, ..., Saturday by 6 in your answer.)

4. 在下圖中， D 以直線連接著等邊三角形 ABC 的頂點，當中 $AB = AD$ 。設 $\angle BDC = \alpha^\circ$ ，求 α 的值。

In the figure below, vertices of equilateral triangle ABC are connected to D in straight line segments with $AB = AD$. If $\angle BDC = \alpha^\circ$, determine the value of α .



Hong Kong Mathematics Olympiad (2013 / 2014)

Final Event 3 (Group)

香港數學競賽 (2013 / 2014)

決賽項目 3 (團體)

除非特別聲明，答案須用數字表達，並化至最簡。

Unless otherwise stated, all answers should be expressed in numerals in their simplest forms.

1. 求乘積 $\left(1 - \frac{1}{2^2}\right)\left(1 - \frac{1}{3^2}\right) \cdots \left(1 - \frac{1}{10^2}\right)$ 的值。

Determine the value of the product $\left(1 - \frac{1}{2^2}\right)\left(1 - \frac{1}{3^2}\right) \cdots \left(1 - \frac{1}{10^2}\right)$.

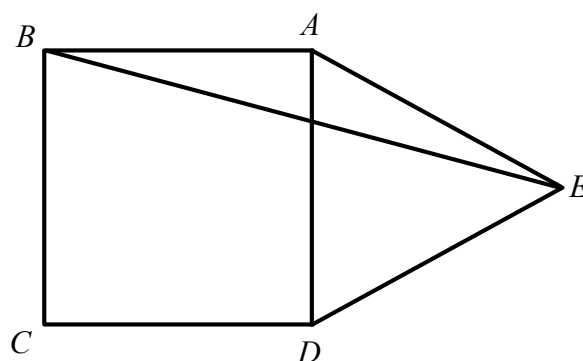
2. 求和 $\frac{1}{\log_2 100!} + \frac{1}{\log_3 100!} + \frac{1}{\log_4 100!} + \cdots + \frac{1}{\log_{100} 100!}$ 的值，當中 $100! = 100 \times 99 \times 98 \times \cdots \times 3 \times 2 \times 1$ 。

Determine the value of the sum $\frac{1}{\log_2 100!} + \frac{1}{\log_3 100!} + \frac{1}{\log_4 100!} + \cdots + \frac{1}{\log_{100} 100!}$, where

$100! = 100 \times 99 \times 98 \times \cdots \times 3 \times 2 \times 1$.

3. 在下圖中， $ABCD$ 是一個正方形， ADE 是一個等邊三角形，且 E 是正方形 $ABCD$ 外的一點。設 $\angle AEB = \alpha^\circ$ ，求 α 的值。

In the figure below, $ABCD$ is a square, ADE is an equilateral triangle and E is a point outside of the square $ABCD$. If $\angle AEB = \alpha^\circ$, determine the value of α .



4. 把不同的非零個位數填進下表白色的正方格內，使所有橫、直的等式均成立。求 α 。

Fill the white squares in the figure below with distinct non-zero digits so that the arithmetical expressions, read both horizontally and vertically, are correct. What is α ?

	÷		=	
+		×		
	+		=	α
=		=		

Hong Kong Mathematics Olympiad (2013 / 2014)

Final Event 4 (Group)

香港數學競賽 (2013 / 2014)

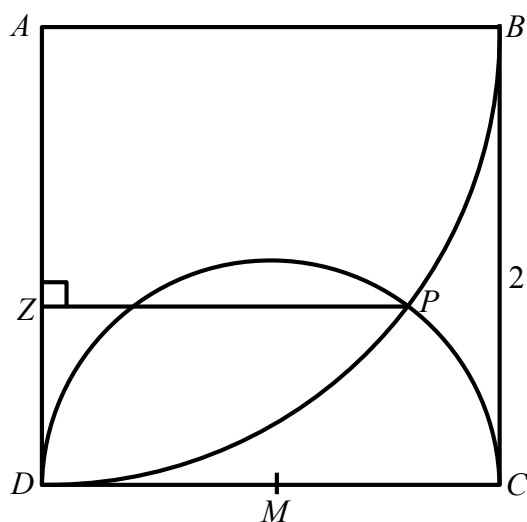
決賽項目 4 (團體)

除非特別聲明，答案須用數字表達，並化至最簡。

Unless otherwise stated, all answers should be expressed in numerals in their simplest forms.

1. 在下圖中， $ABCD$ 是一個邊長為 2 的正方形。先以 A 為圓心畫出弧 BD ，再以 CD 的中點 M 為圓心從 C 到 D 畫出一個半圓。弧 BD 和弧 DC 相交於 P 。求 P 與 AD 的最短距離，即 PZ 的長度。

In the figure below, $ABCD$ is a square of side length 2. A circular arc with centre at A is drawn from B to D . A semicircle with centre at M , the midpoint of CD , is drawn from C to D and sits inside the square. Determine the shortest distance from P , the intersection of the two arcs, to side AD , that is, the length of PZ .



2. 若 $x = \frac{\sqrt{5}+1}{2}$ 及 $y = \frac{\sqrt{5}-1}{2}$ ，求 $x^3y + 2x^2y^2 + xy^3$ 的值。

If $x = \frac{\sqrt{5}+1}{2}$ and $y = \frac{\sqrt{5}-1}{2}$, determine the value of $x^3y + 2x^2y^2 + xy^3$.

3. 若 a, b, c 及 d 是不同的個位數，且

$$\begin{array}{r} a\ a\ b\ c\ d \\ -\ d\ a\ a\ b\ c \\ \hline 2\ 0\ 1\ 4\ d \end{array}$$

求 d 的值。

If a, b, c and d are distinct digits and

$$\begin{array}{r} a\ a\ b\ c\ d \\ -\ d\ a\ a\ b\ c \\ \hline 2\ 0\ 1\ 4\ d \end{array},$$

determine the value of d .

4. 求方程 $x^4 + (x-4)^4 = 32$ 所有實根的乘積。

Determine the product of all real roots of the equation $x^4 + (x-4)^4 = 32$.